Common Errors and Misconceptions in Column Addition and Subtraction

Column method for addition and subtraction builds on pupils understanding of place value and different strategies including knowledge of number bonds within 20 and the 'make ten strategy'. One key misconception that pupils may have when solving column addition and subtraction is considering each digit as a separate number rather than as a representation of the number of tens or ones.

Below are some examples of common errors and misconceptions that you may observe.

Common errors and misconceptions in column addition

Pupils may fail to understand the place value of the digits in the calculation.

For $245 + 33 = \square$ they may write:

\[
\begin{array}{ccc}
\text{hundreds} & \text{tens} & \text{ones} \\
2 & 4 & 5 \\
+ & 3 & 3 \\
\hline \\
5 & 7 & 5
\end{array}
\]

Representing each number in the calculation with Dienes, in a place value grid, will help pupils to address this misconception. Alongside using the Dienes and a place value grid, to model solving calculations, demonstrate turning calculations from the horizontal into the vertical format and explain what you are doing in terms of the place value of each digit.

Pupils may add a column where it does not exist.

For $34 + 18 = \square$ they may write: For $277 + 142 = \square$ they may write:

\[
\begin{array}{ccc}
\text{tens} & \text{ones} \\
3 & 4 \\
+ & 1 & 8 \\
\hline \\
4 & 1 & 2
\end{array}
\]

\[
\begin{array}{ccc}
\text{hundreds} & \text{tens} & \text{ones} \\
2 & 7 & 7 \\
+ & 1 & 4 & 2 \\
\hline \\
3 & 1 & 1 & 9
\end{array}
\]

This shows that the pupil needs to work on their understanding of the place value columns and their ability to use regrouping. They need to develop their knowledge that 12 ones is equal to 1 ten and 2 ones. Modelling the regrouping process with Dienes in a place value grid, whilst allowing pupils to follow the same process, will help address this.
Common errors and misconceptions in column addition

Pupils may reverse the digits when they regroup.
For $45 + 18 = \underline{\phantom{0}00}$ they may write:

<table>
<thead>
<tr>
<th>tens</th>
<th>ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>+</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
</tr>
</tbody>
</table>

For $273 + 72 = \underline{\phantom{0}00}$ they may write:

<table>
<thead>
<tr>
<th>hundreds</th>
<th>tens</th>
<th>ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>+</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>

Modelling the regrouping process using the Dienes and a place value grid will help pupils to overcome this. For the first example you could show with Dienes that $5 + 8$ is equal to 13 ones, which is the same as 1 ten and 3 ones. This will help pupils to see that they need to move 1 lot of ten to the tens column, represented by a 1, whilst the 3 represents 3 ones and, therefore, goes in the ones column.

Pupils may get the regrouping process correct but forget to add what they have regrouped.
For $57 + 34 = \underline{\phantom{0}00}$ they may write:

<table>
<thead>
<tr>
<th>tens</th>
<th>ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>+</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
</tr>
</tbody>
</table>

For $238 + 181 = \underline{\phantom{0}00}$ they may write:

<table>
<thead>
<tr>
<th>hundreds</th>
<th>tens</th>
<th>ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>+</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>9</td>
</tr>
</tbody>
</table>

Again using Dienes and a place value grid alongside the solving the calculation will help to address this. It allows pupils to concretely regroup 10 ones as a ten and move it in the tens column (or 10 tens as 1 hundred). When they then count the Dienes to calculate the total they will see that they include the quantity that they have regrouped.

As pupils move into working with 3-digit numbers, they may not understand that the hundreds column exists when the numbers in the original calculation do not contain any hundreds.
For $58 + 71 = \underline{\phantom{0}00}$ they may write:

<table>
<thead>
<tr>
<th>tens</th>
<th>ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>+</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
</tbody>
</table>

This demonstrates that the pupil needs to work on their knowledge of the place value columns and the process of regrouping from tens to hundreds. For this example you would show that 12 tens are equal to 1 hundred and 2 tens using Dienes.
Common errors and misconceptions in column subtraction

Pupils may subtract the smaller digit in a column from the larger digit, regardless of which appears in the top line. This can arise if they hear that, “for subtraction you always start with the bigger number and subtract the smaller number”.

For $65 - 27 = \square$ they may write: For $238 - 146 = \square$ they may write:

\[
\begin{array}{c c}
\text{tens} & \text{ones} \\
6 & 5 \\
\underline{+} & \underline{2} \\
4 & 2
\end{array}
\quad
\begin{array}{c c c}
\text{hundreds} & \text{tens} & \text{ones} \\
2 & 3 & 8 \\
\underline{+} & \underline{1} & \underline{4} & \underline{6} \\
1 & 1 & 2
\end{array}
\]

This shows that the pupil is not thinking about the calculation as a whole, but treating each column as a separate subtraction in its own right. The pupil needs to work on their understanding that numbers can be partitioned in different ways. For the first example above, they would need develop their understanding that 67 is equal to 60 + 7 but that it is also equal to 50 + 17. Modelling the regrouping process with Dienes demonstrates this.

Pupils may try to record the regrouping process in the written form without understanding what the process represents conceptually. This may result in the following type of errors.

For $72 - 34 = \square$ they may write:

\[
\begin{array}{c c}
\text{tens} & \text{ones} \\
\underline{6} & 2 \\
\underline{+} & \underline{3} & \underline{4} \\
3 & 2
\end{array}
\quad
\begin{array}{c c}
\text{tens} & \text{ones} \\
\underline{7} & \underline{12} \\
\underline{+} & \underline{3} & \underline{4} \\
4 & 8
\end{array}
\]

This demonstrates why it is vital that you so not rush in to the abstract recording when teaching the column method for subtraction. Pupils must use Dienes and place value grids to concretely experience the regrouping process in subtraction, in order to develop a secure conceptual understanding, before recording it in the abstract form. Expose the pupils to the abstract alongside them carrying out the regrouping process concretely.
Common errors and misconceptions when calculations involve numbers with zero as a place holder

If the numbers in the addition calculation contain zero as a place holder pupils may not think about the calculation in its entirety. Instead they see each column as a separate addition, to which they then apply misconceptions regarding adding zero.

For $205 + 174 = \underline{\hspace{2cm}}$

\begin{array}{ccc}
\text{hundreds} & \text{tens} & \text{ones} \\
2 & 0 & 5 \\
+ & 1 & 7 \\
\hline \\
& 3 & 0 \\
\end{array}

Similar errors can also occur for subtraction involving numbers with zero as a place holder. Pupils may not think about the calculation in its entirety and instead see each column as a separate subtraction, to which they then apply misconceptions regarding subtracting from zero.

For $80 - 45 = \underline{\hspace{2cm}}$ they may write: For $306 - 164 = \underline{\hspace{2cm}}$ they may write:

\begin{array}{ccc}
\text{tens} & \text{ones} \\
8 & 0 \\
- & 4 \\
\hline \\
4 & 0 \\
\end{array}

\begin{array}{ccc}
\text{hundreds} & \text{tens} & \text{ones} \\
3 & 0 & 6 \\
- & 1 & 6 \\
\hline \\
2 & 6 \\
\end{array}

Here pupils need to develop their understanding of zero as a place holder. They need experience of representing numbers with zero as a place holder using Dienes in a place value chart, for example seeing that 205 represents 2 hundreds and 5 ones. They need to be encouraged to still think about the calculation as a whole. They also need to develop their understanding of what happens to a number when you add or subtract zero.

The presence of zero of a place holder can also lead to errors if the subtraction involves regrouping:

For $204 - 76 = \underline{\hspace{2cm}}$ they may write:

\begin{array}{ccc}
\text{hundreds} & \text{tens} & \text{ones} \\
2 & 9 & 1 \\
- & 7 & 6 \\
\hline \\
2 & 2 & 8 \\
\end{array}

These types of errors illustrate that the pupils needs to develop thier understanding of zero as a place holder. In this example they have perceived the zero to represent 10, and tried to regroup it (seeing it as 10 ones, only one of which they have moved to the ones column). They need to see that the zero means that zero tens are present to realise that they need to move on to the hundreds column and regrouping one hundred as 10 tens.