

# Mathematics mastery primary conference

Workshop

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## Aims:

To explore, in the context of the new National Curriculum,

- expectations and implications, particularly in relation to differentiation
- characteristics of effective teaching and learning in the context of mastery

# The National Curriculum for mathematics aims to ensure that all pupils:



- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

# The National Curriculum: expectations



The NC states:

The expectation is that the majority of pupils will move through the programmes of study at broadly the same pace.

However, decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress to the next stage.

Pause for thought ...

... what implications does the expectation, 'the majority of pupils will move through the programmes of study at broadly the same pace', have for lesson planning and teaching?

Does this imply a re-interpretation of what 'differentiation' means?

Discuss!



# Challenge through depth

1. Work out

$$(999 - 99 + 9) \div 9$$

Can you do it another way?

<http://www.ukmt.org.uk>

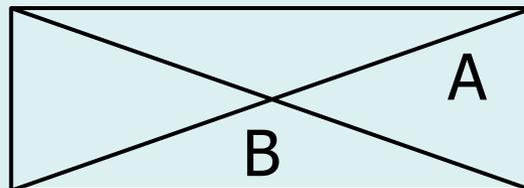
(junior challenge 2014)

2. P, Q, R, S and T represent single digits in this subtraction.

$$\begin{array}{r} 7\ Q\ 2\ S\ T \\ -\ P\ 3\ R\ 9\ 6 \\ \hline 2\ 2\ 2\ 2\ 2 \end{array}$$

What is the value of  $P + Q + R + S + T$  ?

3. What is the ratio of the areas of triangles A and B?



# The National Curriculum: differentiation?

The NC states:

Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content.

Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on.

Developing fluency ...

... what does good consolidation look like?



# Practice makes perfect?

Compare these two multiplication exercises.

Which supports the development of fluency better? Why?

$8 \times 5 =$	$8 \times 3 =$	$9 \times 4 =$	$9 \times 4 =$	$7 \times 9 =$	$1 \times 4 =$
$2 \times 8 =$	$5 \times 2 =$	$3 \times 9 =$	$6 \times 3 =$	$6 \times 8 =$	$8 \times 5 =$
$1 \times 1 =$	$3 \times 8 =$	$2 \times 5 =$	$9 \times 2 =$	$7 \times 7 =$	$4 \times 6 =$

$2 \times 3 =$	$6 \times 7 =$	$9 \times 8 =$
$2 \times 30 =$	$6 \times 70 =$	$9 \times 80 =$
$2 \times 300 =$	$6 \times 700 =$	$9 \times 800 =$
$20 \times 3 =$	$60 \times 7 =$	$90 \times 8 =$
$200 \times 3 =$	$600 \times 7 =$	$900 \times 8 =$

# The NC: a mastery curriculum



- An expectation that all pupils can and will achieve.
- The large majority of pupils progress through the curriculum content at the same pace. Differentiation emphasises deep knowledge and individual support/intervention.
- Teaching is underpinned by methodical curriculum design, with units of work that focus in depth on key topics. Lessons and resources are crafted carefully to foster deep conceptual and procedural knowledge.
- Practice and consolidation play a central role. Well-designed variation builds fluency and understanding of underlying mathematical concepts in tandem.
- Teachers use precise questioning to check conceptual and procedural knowledge. They assess in lessons to identify who requires intervention so that all pupils keep up.

# The NC: a mastery curriculum



- These points are taken from the NCETM's paper on Mastery [www.ncetm.org.uk/public/files/19990433/Developing\\_mastery\\_in\\_mathematics\\_october\\_2014.pdf](http://www.ncetm.org.uk/public/files/19990433/Developing_mastery_in_mathematics_october_2014.pdf)
- A mastery curriculum often involves whole-class teaching, with all pupils being taught the same concepts at the same time. Small-group work typically involves challenge through greater depth for the more able and support with grasping concepts and methods for less-able pupils.
- 'Variation' in set exercises is also known as 'intelligent practice'. Such exercises usually concentrate on the same topic/method/concept but vary in how the questions are presented, often in ways that expose the underlying concept or mathematical structure, and make pupils think deeply for themselves.

Developing fluency ...

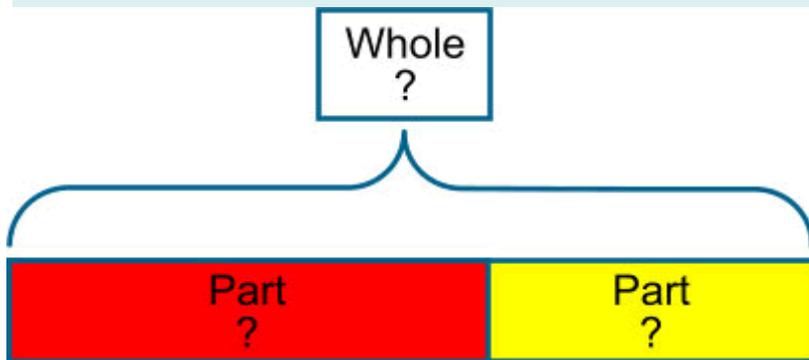
... securing depth of understanding



# Models, images and practical apparatus

All of these play an important part in supporting pupils' conceptual understanding and reasoning skills.

Can you name these?



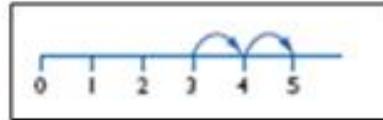
Flexibility with different representations is an important element of fluency.

# Resources to help build concepts

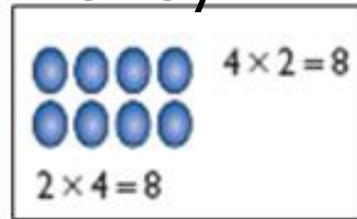
Numicon



number line



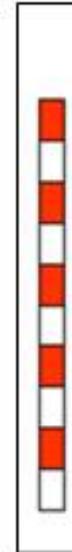
array



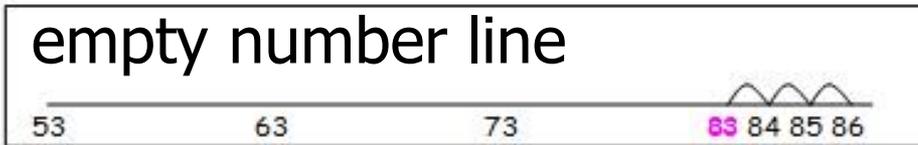
geoboard



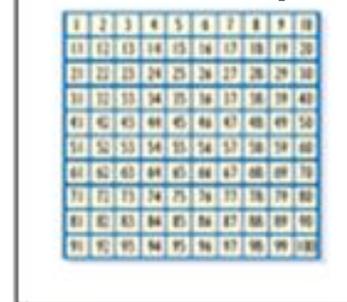
counting stick or metre rule



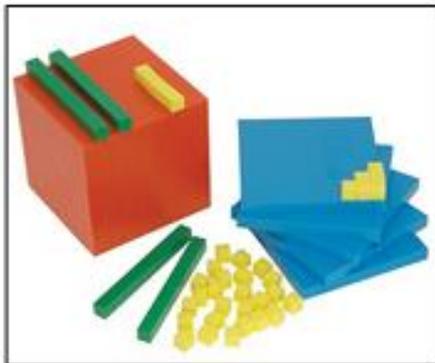
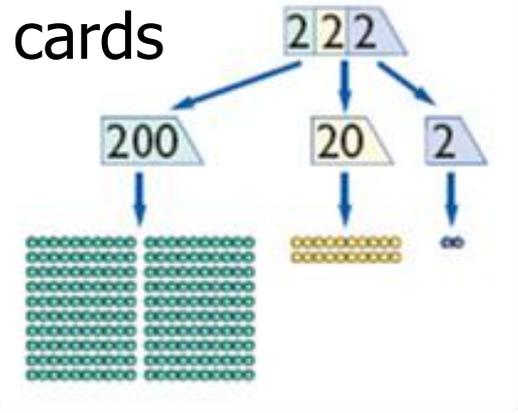
empty number line



hundred square

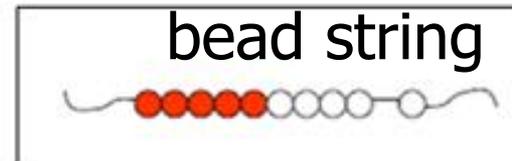


place value



Dienes blocks  
base-ten blocks

bead string



Developing fluency ...

... securing understanding and proficiency



# Video of part of a lesson



## Column subtraction

[www.ncetm.org.uk/resources/40532](http://www.ncetm.org.uk/resources/40532)

When watching the video, think about:

- the progression in the mathematics the pupils are learning
- how the teacher and pupils use practical equipment and/or images
- features of the teaching that support and deepen pupils' conceptual development through the design of the activity, the teacher's explanations and/or questioning

Developing good problem solving ...  
... including challenging the more able



# Problems and puzzles



- Problems do not have to be set in real-life contexts.
- Providing a range of puzzles and other problems helps pupils to reason strategically to:
  - find possible ways into solving a problem
  - sequence an unfolding solution to a problem
  - use recording to help their thinking about the next step.
- It is particularly important that teachers and teaching assistants stress such reasoning, rather than just checking whether the final answer is correct.
- All pupils need to learn how to solve problems from the earliest age – the EYFS early learning goals also include problem solving.

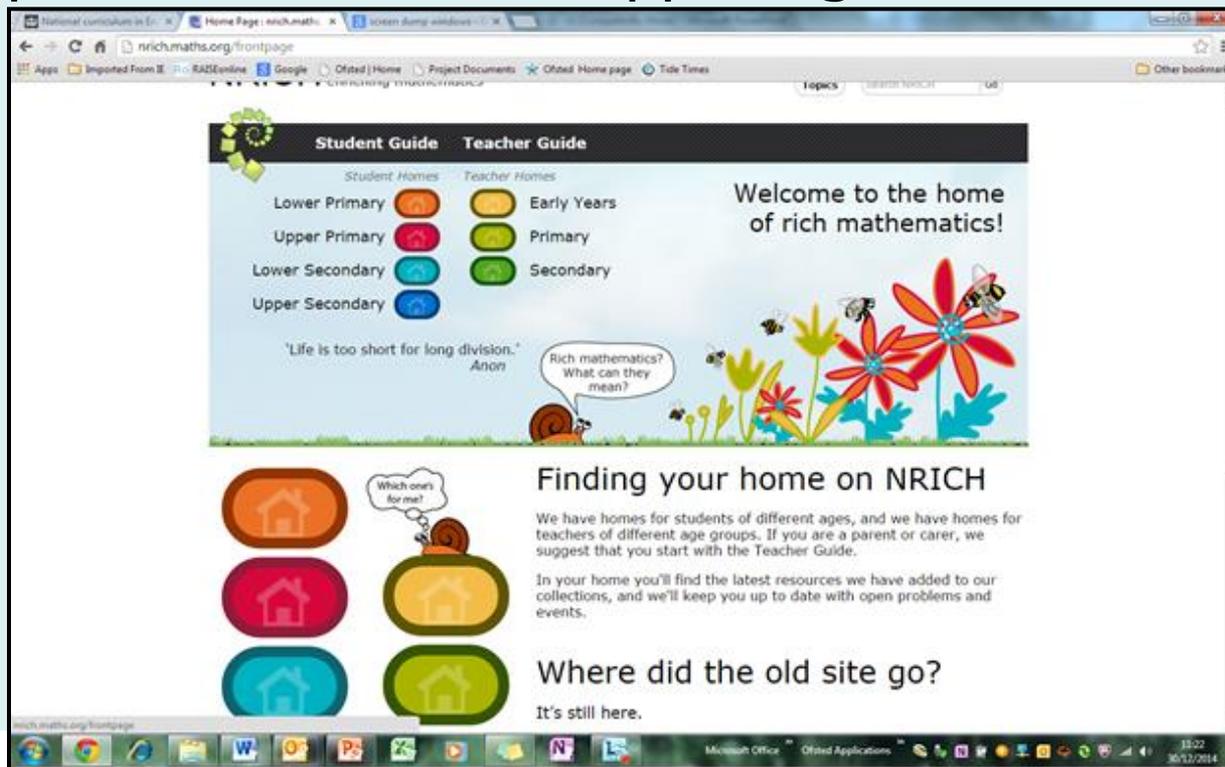
# Common weaknesses in teaching problem solving



- Pupils are expected to acquire problem-solving skills without them being made explicit. Lesson objectives and planning tend to focus on content rather than specific problem-solving skills.
- Teachers/TAs are too quick to prompt pupils, focusing on getting 'the answer' – pupils need to build their confidence and skills in solving problems, so that they can apply them naturally in other situations.
- When problems are set, teachers do not use them well enough to discuss with pupils alternative approaches and why one is more elegant than another.
- Problems for high attainers involve harder numbers rather than more demanding reasoning and problem-solving skills.

# Problem solving: [nrich.maths.org](http://nrich.maths.org)

- The nrich website is a good source for problems.
- It includes printable resources, notes for teachers and solutions written by pupils.
- Each problem has been mapped against the new NC.



The problem, 'Forwards add backwards' is shown on the next slide.

Spend a couple of minutes on it.

- Which pupils in a class might it be suitable for? Why?

Answers and explanation at

[nrich.maths.org/11111](http://nrich.maths.org/11111)



## 'Forwards add backwards'

The number 726 can be formed by adding a 3-digit number with its reversal:  $462 + 264 = 726$ , for example.

- Can you find the other two ways of making 726 in this way?
- Can you find the three ways to do this for 707 and 766?

Which ten numbers between 700 and 800 can be formed from a number plus its reversal?

- What common property do they have?  
Can you explain why?

How many numbers between 300 and 400 can be formed from a number plus its reversal?

How about between 800 and 900?

Improving problems ...

... what is the area of this rectangle?



Adapt this question to encourage pupils to think harder about how to solve it, and to develop better their problem-solving skills and conceptual understanding of area of a rectangle.



# Building variety in problem solving



Straightforward problems can be adapted to create more opportunities for reasoning and for learning about different problem-solving strategies, by:

- removing intermediate steps
- reversing the problem
- making the problem more open
- asking for all possible solutions
- asking why, so that pupils explain
- asking directly about a mathematical relationship.

Developing reasoning ...

... research by Terezinha Nunes (2009) identified the ability to reason mathematically as the most important factor in a pupil's success in mathematics.

*Development of Maths Capabilities  
and Confidence in Primary School*

<http://dera.ioe.ac.uk/11154/1/DCSF-RR118.pdf>



# Reasoning

- Reasoning is integral to the development of conceptual understanding and problem-solving skills.
- Of the three National Curriculum aims, it is the least well developed currently.
- Not all classrooms have a positive ethos that encourages learning from mistakes.
- Tasks are not used well enough to develop reasoning.
- Talk often focuses on the 'how' rather than the 'why', 'why not', and 'what if' in:
  - teachers' explanations and questions
  - pupils' responses.

# NCETM progression maps



- The National Centre for Excellence in the Teaching of Mathematics (NCETM) has produced progression maps for different strands of mathematics within the NC at KS1-3 [www.ncetm.org.uk/resources/44672](http://www.ncetm.org.uk/resources/44672)
- It has added questions to each section within the strands to encourage discussion and reasoning. These include:
  - what do you notice?
  - true or false?
  - odd one out?
  - do, then explain
  - spot the mistake
  - give an example of ...
  - continue the pattern
  - convince me/prove it
- Such questions are useful to encourage all pupils to think and reason, but also good for challenging the more able.